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Book Review: *Small Worlds. The Dynamics of Networks Between Order and Randomness*

Small Worlds. The Dynamics of Networks Between Order and Randomness. Duncan J. Watts, Princeton University Press, Princeton, New Jersey, 1999.

In 1976, at a Bicentennial Physics lecture, Elliot W. Montroll the famed statistical physicist, recounted how he was 5 handshakes away from Benjamin Franklin. Duncan Watts, the author of Small Worlds, remembers his father telling him that most Americans are only 6 handshakes away from the President. Mathematicians have the Erdos numbers ranking how many rungs in a hierarchy they are away from being an Erdos co-author. I thought I didn't have an Erdos number, but quickly discovered that through co-authoring a paper with the Book Editor of this journal that my Erdos number is 3 because his is 2. Then there is the Kevin Bacon Game evolved around demonstrating that any film actor can be linked within 6 connections to a Kevin Bacon film. There is also the belief that one get can from one web page to any other web page using only about 10 clicks. All of these anecdotal stories imply that social networks are "small world" networks with only a few connections relating most sites. A regular lattice nearest neighbor connection scheme would place distance sites a great many connections apart. Small world networks are somewhere between perfect lattice structures and random networks.

It is not surprising that there is a whole mathematics of networks related to graph theory. Watts reviews classical known results, e.g., the Erdos–Renyi theorem: Almost any random graph with n nodes and greater than $n/(2 \ln n)$ edges will be connected. One quickly realizes that large networks are complex systems in the sense that there are too many possible connections to enumerate and analyze. Most questions about networks cannot be answered exactly analytically, but one needs to appeal to scaling laws, typical behaviors, simulations, and guesses. Watts addresses myriad questions. What is a cluster on a network? What is the average cluster size?

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Does this undergo a phase transition as the connectivity increases? What else undergoes a phase transition? What is the average distance between randomly chosen sites? What is the average number of connections between randomly chosen sites? What is the best way to characterize a network? When is a network a small world? Watts chooses three interesting networks to examine and compare detail. The physically real regional Western States Power Grid, the biological neuron system of the worm C. Elegans, and the relational Kevin Bacon Graph connections between movie actors. Comparisons are made among these networks. The power grid had 225,226 nodes while the worm has 302 neurons. Other measures are the average number of connections to a site, the average of number of connections between distant sites. The notation for all of these, and other quantities can be confusing. This book is in serious need of a glossary. The second half of the book investigates other types of networks involving dynamical behaviors, including the spreading of disease, cellular automata, games on graphs including the Prisoner's dilemma, and oscillator arrays.

Watts has covered several types of networks (physical and relational) and many aspects of networks and reviewed relevant parts of graph theory. He has shown that many different types of real networks have a small world character. This book is a great starting place to get involved with the nature of networks.

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